

**Objective:**

The student will gain the knowledge and skills necessary to pass the College Board AP Calculus BC exam. Previous requirement is that this student should have taken the Calculus AB course and received a 3 on the AP Calculus AB exam.

**Course Content:**

Main topics include limits, derivatives, integration and graphing of these functions from the AB exam. You should have received a 3 on the AP Calculus AB exam. In addition, there will be further concepts of differentiation and integration as well as applications of parametric equations, polar equations, sequences, series, vectors (velocity & acceleration), L’hopital’s Rule and Euler method. Students will also have to solve problems with AND without the calculator. See following pages for more descriptive layout. Students are expected to solve problems in a variety of ways as required by the AP Calculus program: graphically, numerically, analytically and verbally.

**Materials:**

Textbook: *Calculus 6<sup>th</sup> ed.* By Stewart  
TI – 83 Silver Edition Calculator (or any TI-83 Plus calculator)  
3 –Ring Binder (1.5 or 2 inch) w/ paper and dividers  
Highlighters  
Pencil w/ eraser

**Classroom Procedures:**

1. **No computers are to be used while teacher is not in class, i.e. substitute.**
2. **Computers are to be used ONLY for educational purposes therefore there are no emails, games, improper websites, etc.**
3. Be seated **PRIOR** to bell.
4. Be prepared.
5. Wait to use the bathroom or other places while teacher is lecturing.
6. 3 passes per marking period.
7. No food or drink in the classroom. Gum is OK until it becomes a nuisance.
8. ALL homework assignments are to be completed on an **INDIVIDUAL** basis unless otherwise stated.
9. Have FUN!!!

**Absences:**

**Excused** – must have an admit slip and you have 1 day to make up all work. You can find all previous work in the absent folders.

**Unexcused** – you may not turn in the work for a grade but you are responsible for all information on missed assignments.

**Lates:**

**Excused** – must have a legal pass. This should **NOT** become a habit.

**Unexcused Late** – more than 15 minutes is classified as a cut!

**Early Dismissals and Tardy Excused:**

If you miss class but are present in school, you are still responsible for turning in any work due on this day. Also, you are responsible to get any assignments that are given on that day. You are in school, you can come see me. This includes all sporting events and any other extracurricular activities. In the event of missing a test, you should schedule with me **AHEAD** of time. **MORE** than one day's notice is required.

**Grading:**

Tests

Quizzes

Homework/Calculator Assignments

**Method of Grade Computation:**
$$\% \text{ Grade} = \frac{\text{accumulated points acquired by student}}{\text{total possible points}}$$
**Grade Scale:**

A = 93 – 100

B = 85 – 92

C = 77 – 84

D = 70 – 76

F = Below 70

**AP Calculus BC Exam: Wednesday, May 5, 2010**

## **Course Description**

Calculus BC is designed to develop the understanding of the concepts of calculus and provide experience with its methods and applications. The course represents a multi-representational approach to calculus, with concepts, results and problems expressed geometrically, numerically, analytically and verbally. The connections among these representations are important. This course is a follow-up to the AP Calculus AB, thus students are expected to have a base understanding of limits, derivatives and integrals.

Broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems or problem types. Technology should be used regularly to reinforce the relationships among multiple representations of functions, to confirm written work, to implement experimentation and to assist in interpreting results.

Through the use of unifying themes of derivatives, integrals, limits, approximations and applications, and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions from pre-calculus.

## **Content (+ is the additions to the AB for the BC)**

### **1. Functions, Graphs and Limits**

**Analysis of graphs.** With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

#### **Limits of functions (including one-sided limits)**

- 1 An intuitive understanding of the limiting process.
- 2 Calculating limits using algebra.
- 3 Estimating limits from graphs or tables of data.

#### **Asymptotic and unbounded behavior**

- 1 Understanding asymptotes in terms of graphical behavior.
- 2 Describing asymptotic behavior in terms of limits involving infinity.
- 3 Comparing relative magnitudes of functions and their rates of change.

#### **Continuity as a property of functions**

- 1 An intuitive understanding of continuity.
- 2 Understanding continuity in terms of limits.
- 3 Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

- 1 **Parametric, polar and vector functions** The analysis of planar curves includes those given in parametric form, polar form and vector form.

## 2. Derivatives

### Concept of the derivative

- 1 Derivative presented graphically, numerically and analytically.
- 2 Derivative interpreted as an instantaneous rate of change.
- 3 Derivative defined as the limit of the difference quotient.
- 4 Relationship between differentiability and continuity.

### Derivative at a point

- 1 Slope of a curve at a point.
- 2 Tangent line to a curve at a point and local linear approximation.
- 3 Instantaneous rate of change as the limit of average rate of change.
- 4 Approximate rate of change from graphs and tables of values.

### Derivative as a function

- 1 Corresponding characteristics of graphs of  $f$  and  $f'$ .
- 2 Relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$ .
- 3 The Mean Value Theorem and its geometric consequences.
- 4 Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

### Second derivative

- 1 Corresponding characteristics of the graphs of  $f$ ,  $f'$ , and  $f''$ .
- 2 Relationship between the concavity of  $f$  and the sign of  $f''$ .
- 3 Points of inflection as places where concavity changes.

### Applications of derivatives

- 1 Analysis of curves.
- 2 Analysis of planar curves given in parametric form, polar form and vector form, including velocity and acceleration.
- 2 Optimization, both absolute (global) and relative (local) extrema.
- 3 Modeling rates of change including related rates problems.
- 4 Use of implicit differentiation to find the derivative of an inverse function.
- 5 Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration.
- 6 Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.
- 1 Numerical solution of differential equations using Euler's method.
- 2 L'Hopital's Rule, including its use in determining limits and convergence of improper integrals and series.

### Computation of derivatives

- 1 Knowledge of derivatives of basic function, including power, exponential, logarithmic, trigonometric and inverse trigonometric functions.
- 2 Basic rules for the derivative of sums, products and quotients of functions.

- 3 Chain rule and implicit differentiation.
- 1 Derivatives of parametric, polar and vector functions.

### 3. Integrals

#### Interpretations and properties of definite integrals

- 1 Computation of Riemann sums using left, right and midpoint evaluations.
- 2 Definite integral as a limit of Riemann sums over equal subdivisions.
- 3 Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

- 4 Basic properties of definite integrals (examples include additivity and linearity).
- 1 **Applications of integrals** Appropriate integrals are used in a variety of applications to model physical, biological or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the method its limit as a definite integral. To provide a common foundations, specific applications should include using the integral of a rate of change to give accumulated change, finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line and the length of a curve (including a curve given in parametric form.)

#### Fundamental Theorem of Calculus

- 1 Use the Fundamental Theorem of Calculus to evaluate definite integrals.
- 2 Use the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

#### Techniques of antidifferentiation

- 1 Antiderivatives following directly from derivatives of basic functions
- 2 Antiderivatives by substitution of variables (including change of limits for definite integrals), parts and simple partial fractions (nonrepeating linear factors only)
- 2 Improper integrals (as limits of definite integrals)

#### Applications of Antidifferentiation

- 1 Finding specific antiderivatives using initial conditions, including motion along a line
- 2 Solving separable differential equations and using them in modeling (in particular, studying the equation  $y' = ky$  and exponential growth).
- 1 Solving logistic differential equations and using them in modeling.

#### Numerical Approximations to Definite Integrals

- 1 Riemann sum
- 2 Trapezoidal rule

**+ 4. Polynomial Approximations and Series**

+ **Concept of series** A series is defined as a sequence of partial sums and convergence is defined in terms of the limit of the sequence of partial sums. Technology will be used to explore convergence and divergence.

+ **Series of constants**

- 1 Motivating examples, including decimal expansion
- 2 Geometric series with applications
- 3 The harmonic series
- 4 Alternating series with error bound
- 5 Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of  $p$ -series.
- 6 The ratio test for convergence and divergence
- 7 Comparing series to test for convergence or divergence

+ **Taylor Series**

- 1 Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve)
- 2 Maclaurin series and the general Taylor series centered at  $x = a$
- 3 Maclaurin series for functions  $e^x$ ,  $\sin x$ ,  $\cos x$  and  $\frac{1}{1-x}$
- 4 Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation and the formation of new series from known series
- 5 Functions defined by power series
- 6 Radius and interval of convergence of power series
- 7 Lagrange error bound for Taylor polynomials

**Course Materials**

Finney, Ross L., Franklin D. Demana, Bert K. Watts and Daniel Kennedy. Calculus: A Complete Course, 2<sup>nd</sup> ed., Reading, MA, Addison-Wesley, 2000.  
 Stewart, James. Calculus 6<sup>th</sup> ed., Belmont, CA: Thomson Learning Inc., 2003.

**Supplemental Materials**

Larson, Ron, Robert P. Hostetler, Bruce H. Edwards and David E. Heyd. Calculus with Analytical Geometry, 7<sup>th</sup> ed., New York: Houghton Mifflin Company, 2002.

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**Assessments:**

There are tests for each unit. Also, there are quizzes for at least every 2 sections. Occasionally, there are quizzes per section.

**Calculator Activity:**

**Goal:** To explore whether a sequence and/or a series converges or diverges using lists.

**Tools:** TI – 83 calculator (or some version)

**Calculator:** Making 3 lists.

List 1 (L1): represents the n value for your sequence and series

List 2 (L2): represents the sequence for that n value

List 3 (L3): represents the series for that n value

Make a list: STAT – EDIT – 1)Edit – Up arrow so that L1 is highlighted.

L1 = should be showing at the bottom

Type 2<sup>nd</sup> STAT (List) – OPS – 5:seq(

Now at the bottom of the list, you should have

L1 = seq(

Type n, n, 1, 50) so that the bottom shows

L1 = seq(n, n, 1, 50)

The column should fill from 1 to 50, respectively.

Do the same for L2, but

L2 = seq(f(n), n, 1, 50)

For L3,

Type 2<sup>nd</sup> STAT – OPS – 6: cumSum(

Complete L3 by typing,

L3 – cumSum(L2) (get L2 by 2<sup>nd</sup> 2, it is above the 2)

**Analysis:**

Find if the series converges for: a)  $a_n = n^2$

b)  $b_n = \frac{1}{n}$

c)  $c_n = \frac{1}{n^3}$

d)  $d_n = \frac{e^n}{n!}$

e)  $f_n = \frac{2}{4n^2 - 1}$

$$f) g_n = \frac{2n^2}{n^2 + 1}$$

Which sequences converge? If so, what is the value? Explain.

Which series converge? If so, what is the value? Explain.